EduTap

MIND MAP

for Instant Revision



Time Value of Money

Time Value of Money

Compounding Frequency.

Annual Compounding

If interest is compounded annually, you earn or pay interest on your principal once a year.

In this case, interest is compounded once a year. So, amount will be calculated as follows -
A = P * (1 + R)^(T),
A = 10,000 * (1 + 0.10)^3, A = 10,000 * (1.10)^3, A ≈ 13,310

Frequent Compounding: When interest is compounded more frequently, you earn or pay interest multiple times within the year

Semi-Annual Compounding

In semi-annual compounding, interest is added or calculated twice a year, typically every six months. Therefore, the compounding frequency over here becomes 2, which means you will get the interest 2 times in a year.

Interest is compounded twice a year, means you will get interest two times in a year, therefore compounding frequency over here is 2. Now, you can use the basic formula, A = P * (1 + R)^(T)---> A = 10,000 * (1 + 0.05)^6, A = 10,000 * (1.05)^6, A ≈ 13,476.78

Quarterly Compounding

It involves adding or calculating interest every three months (four times a year). Therefore, the compounding frequency over here becomes 4, which means you will get the interest 4 times in a year.

here Interest is compounded four times a year, so compounding frequency will change to 4. Now, you can use the basic formula, A = P * (1 + R)^(T)., A = 10,000 * (1 + 0.0.25)^12, A = 10,000 * (1.025)^12-----> A ≈ 13,614.29

Monthly Compounding:

Monthly compounding means that interest is added or calculated every month (twelve times a year). Therefore, the compounding frequency over here becomes 12, which means you will get the interest 12 times in a year.

Continuous Compounding

It represents the highest frequency, where interest is calculated and added continuously.
Formula is A = P * e^(rt). Where A is the Future Value (the total amount including both principal and interest), P is the Principal Amount (the initial sum of money), e is the mathematical constant approximately equal to 2.71 (You have to assume this), r is the Annual Interest Rate (in decimal form), t is the time the money is invested or borrowed for, in years.

Question - You invest Rs. 5,000 in a savings account that offers a continuous compounding interest rate of 6% per year. How much will your investment be worth after 3 years? Using the formula for continuous compounding: A = P * e^(rt)

Let's put the values in the formula now

- A = 5,000 * e^(0.06 * 3)
- A = 5,000 * e^0.18
- Now, calculate e^0.18 (using the approximate value of e = 2.71):
- A ≈ 5,000 * 1.197163
- A ≈ Rs. 5,985.82

Definition

Time Value of Money (TVM) explores the idea that the value of money today is not the same.

It is often said that “money which you hold today is more valuable than the money you hold in future"

This statement essentially means that a sum of money today is worth more than the same sum of money at a future date. It highlights the idea that having money now gives you the ability to use it immediately, invest it, or take advantage of opportunities

Reasons for Time Value of Money

- **Investment Opportunities** — "Investment Opportunities" means that when you have money today, you can use it to explore various ways to make it grow, such as investing in stocks, bonds, or even depositing it in a bank account.
- **Risk and Uncertainties** — "Risk and Uncertainties" refers to the idea that money available today is generally considered less risky than the same amount of money expected in the future.
- **Inflation** — Over time, money loses its buying power due to rising prices. Money today can buy more than the same amount in the future.
- **Consumption:** — Individuals generally prefer current consumption to future consumption

Techniques of Time Value of Money

Present Value (PV)

It measures the current value of an amount of money. Think of it as the money you hold today. PV helps you understand the current worth of a sum of money. For example, if you have Rs. 10,000 with you currently and you want to you invest the same in the bank accounts, then Rs. 10,000 is the PV.

Future Value (FV)

It measures the Future value of an amount of money. It’s like a treasure chest that shows you how much your money can grow over time.

Formula => Future Value = Present Value + Interest

Simple Interest

It is a straightforward way of earning or paying interest on money. It's calculated only on the initial sum, known as the principal. You earn or pay a fixed percentage of the principal, regardless of how much you've earned or paid in the past.

formula (SI) is: SI = (P * R * T) / 100, where, SI = Simple Interest, P = Principal Amount (the initial sum of money), R = Rate of Interest per year (as a percentage), T = Time period in years for which interest is calculated.

You deposit Rs.1,000 @ 5% annual interest rate. How much interest will you earn after 2 years? Answer - SI = 1,000*5*2 / 100 = Rs.100

Compound Interest

It is a method of earning or calculating interest on money that involves not only the initial sum (the principal) but also the interest that accumulates over time.

formula, A = P * (1 + R)^(t), where-> A is the future amount, including both principal and interest, P is the principal amount , R is the annual interest rate (in decimal form), T is the time in years (here T can also be written as n, both the things mean the same)

Imagine you have Rs. 10,000, and you decide to invest it in a savings account with a 10% annual interest rate, compounded annually.
•Year 1: At the end of the first year, your initial principal of Rs. 10,000 earns 10% interest, which is Rs. 1,000. So, you now have Rs. 11,000.
•Year 2: In the second year, you don't just earn interest on the initial Rs. 10,000; you earn it on the new total, which is Rs. 11,000. At a 10% rate, you earn Rs. 1,100 in interest. So, you now have Rs. 12,100.
•Year 3: In the third year, you continue to earn interest on Rs. 12,100, not just the original Rs. 10,000. At 10%, you earn Rs. 1,210. So, your total amount to be received becomes Rs. 13,310

Key Observation - Interest earned via compound interest method is bigger as compared to the simple interest method, given the same principal amount, interest rate, and time period.

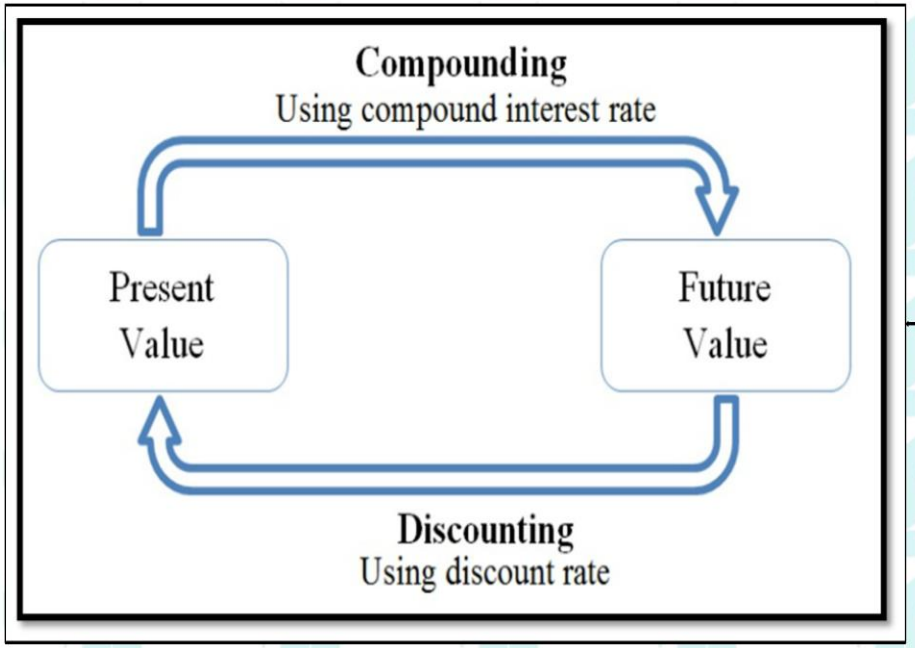
Techniques of Time Value of Money

Discounting

Discounting is like the reverse journey of compounding in the world of finance. While compounding takes your money from Present Value (PV) to Future Value (FV), discounting does the opposite; it takes your money from Future Value (FV) to Present Value (PV)

Here's how it works:

1. Future Value (FV): You have a specific amount of money that you expect to receive or spend at some point in the future. This is your FV.
2. Time Factor: Money in the future is not as valuable as money today because you could be using it or earning interest on it. So, you "discount" it to find out what it's worth today. We will discuss more about it later on.
3. Calculating PV: Discounting helps you find out how much money you need to have today to end up with the FV you expect in the future.

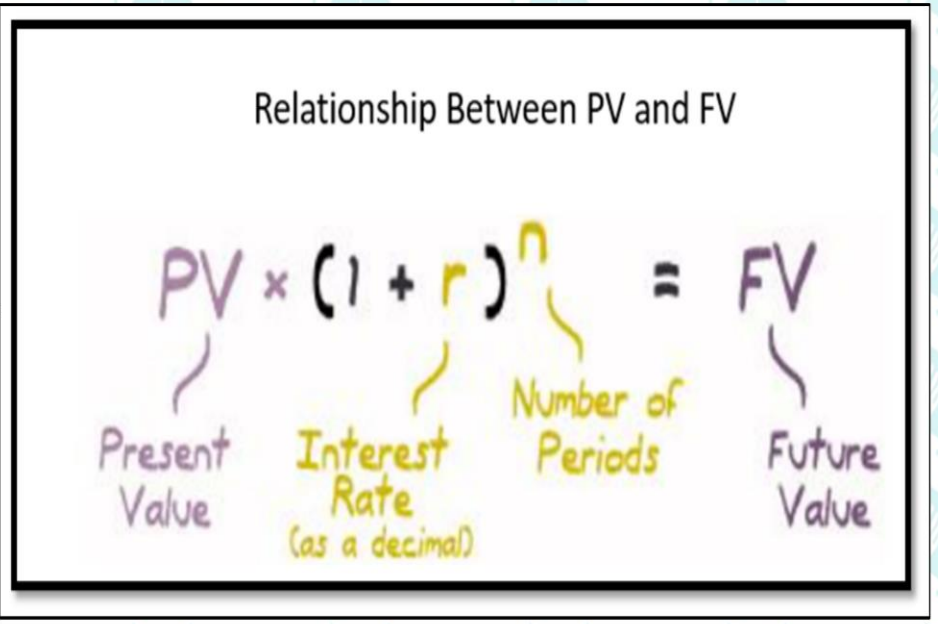


Practical Application of Discounting

You are expecting to receive Rs. 1,000 one year from today. If you want to determine the present value of that amount today, assuming a discount rate of 8%, what is the present value?

The formula remains the same, which is FV = PV * (1 + r)^t . Now, let's calculate it:

- PV = 1,000 / (1 + 0.08)
- PV = 1,000 / 1.08
- PV ≈ Rs. 925.93



Compounding

Compounding is the process whereby interest is credited to an existing principal amount as well as to interest already paid.

Here's how compounding unfolds:

1. Starting Point (PV): You begin with a certain amount of money today. This is your PV, the foundation of your financial journey.
2. Investment: You decide to put that money to work. You invest you money and therefore you will start earning interest on it.
3. Arrival at Future Value (FV): Eventually, after a certain period of time, you reach your destination, which is the Future Value (FV). This is the total amount you end up with, which includes both your initial principal and all the interest accumulated over time.

Practical Application of Compounding

Q-> If you deposit Rs. 25,000 in a bank that offers a 8% annual interest rate on a four-year time deposit, how much would the deposit grow at the end of four years?

To calculate the future value (or the amount), we shall be using our basic formula only, which FV = PV * (1 + r)^t

- FV = 25,000 * (1 + 0.08)^4
- FV ≈ 25,000 * 1.36049
- FV ≈ Rs. 34,012.25

Introduction to Annuities

Calculating the Present Values and Future Values of annuities

Future Value of Ordinary Annuity

Suppose you invest 1000 at the end of each year for next 5 years with interest rate to be 5%. What would be total value of money at the end of 5 years?

Where, C = Cash flow per period, i = interest rate, n = number of payments

$$FV_{Ordinary\ Annuity} = C * \left[\frac{(1+i)^n - 1}{i}\right]$$

Just plug in the values now, you will get

- FV = 1000 [(1+.05)5-1] / .05
- FV = 1000 [1.27628 - 1] / .05
- FV = 1000 [.27628] / .05
- FV = 276.28/.05
- FV = 5525.6

Future Value of Annuity Due

$$FV\ of\ Annuity\ Due = (1+r) \times P\left[\frac{(1+r)^n - 1}{r}\right]$$

$P = Periodic\ Payment$
$r = rate\ per\ period$
$n = number\ of\ periods$

Present Value of Ordinary Annuity

Question - What is the present value of the project which pays cash flows of 1000 at the end of each year for next 5 years assuming interest rate to be 5%?

$$PV_{Ordinary\ Annuity} = C * \left[\frac{1 - (1+i)^{-n}}{i}\right]$$

Apply the values in the formula, you will get

- PV = 1000 * [(1 - (1+.05) -5] / .05
- PV = 4329.47

Present Value Annuity due

For ordinary annuity payments are done at the end of each period. However, for the Annuity Due – Payments are made at the beginning of each period

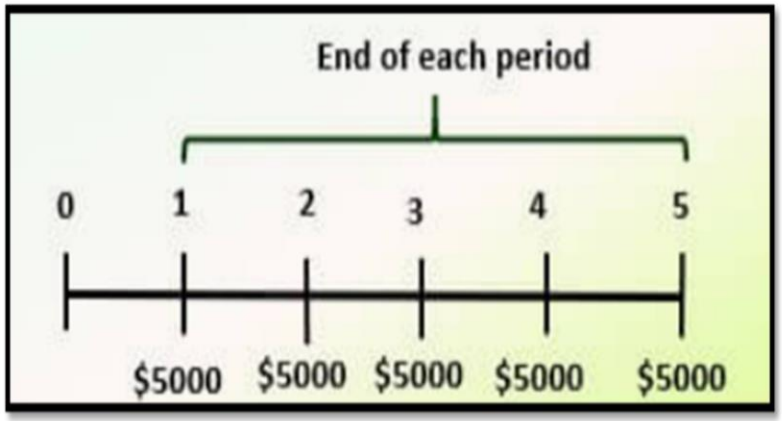
$$PV_{Annuity\ Due} = C * \left[\frac{1 - (1+i)^{-n}}{i}\right] * (1+i)$$

An annuity is like a series of regular payments or receipts that happen at a fixed time interval.

Types of Annuities

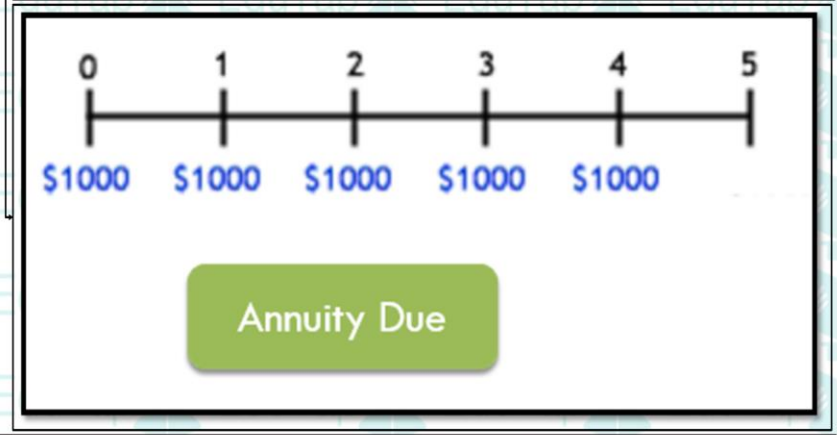
Ordinary Annuity:

An ordinary annuity is a type of annuity where the payments or receipts occur at the end of each period.



Annuity Due

An annuity due, on the other hand, is a type of annuity where the payments or receipts happen at the beginning of each period.



Primary difference between annuity due and ordinary annuity lies in the timing of the amount. In an ordinary annuity, payments/Income occur at the end of each period, while in an annuity due, payments/income take place at the beginning of

Time value of Money

Short cut tricks to calculate the compounding

Rule of 72

By dividing 72 by the annual rate of return, investors can get a rough estimate of how many years

Rule of 114

To estimate how long it takes to triple your money, divide 114 by your expected interest rate (or rate of return)

Rule of 144

To estimate how long it takes to Quadruple your money, divide 144 by your expected interest rate (or rate of return)

Effective Interest Rate

It is also known as the Annual Equivalent Rate (AER) or Annual Percentage Yield (APY), is a measure that reflects the true or effective annual interest rate earned or paid on an investment, loan, or financial product, taking into account the effects of compounding.

In simpler terms, the effective interest rate represents the total interest or return you'll receive or pay over the course of a year, factoring in how often interest is added to the principal amount.

$$EIR = (1+r/m)^{m} - 1$$

where EIR = Effective Rate of Interest

r = Nominal Rate of Interest (Yearly Interest Rate)

m = Frequency of compounding per year

Present Value and Future Value Factors, are often referred to tables which are precomputed values used in finance to simplify the calculation.

Future Value of 1 Factors (FV of 1 factors)

n	1%	2%	4%	8%	12%
2	1.020	1.040	1.082	1.166	1.254
5	1.051	1.104	1.217	1.469	1.762
10	1.105	1.219	1.480	2.159	3.106
12	1.127	1.268	1.601	2.518	3.896
15	1.161	1.346	1.801	3.172	5.474
16	1.173	1.373	1.873	3.426	6.130
24	1.270	1.608	2.563	6.341	15.179

The above table gives you, which represents the present value of 1 Unit for a combination of years (n) and Interest rates (r). These are also called present Value Interest Factors

If you deposit Rs. 25,000 in a bank that offers 8% annual interest rate on a four-year time deposit, how much would the deposit grow at the end of two years?

FV = PV * (1 + r)^t, FV = 25,000 * (1 + 0.08)^2, FV = 25,000 * (1.08)^2, FV ≈ 25,000 * 1.1664 , FV ≈ Rs. 29,160

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