EduTap

SUMMARY SHEET



Time Value of Money

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Important Points

1. This Summary Sheet shall only be used for Quick Revision purpose, and revision should happen after you have completely read the Complete Notes.

2. For Building Concepts along with examples/concept checks you should rely completely only on Cocept Notes as provided

3. It would be useful to go through this Summary sheet just before the exam or before any Mock Test.

4. Questions in the exam are concept-based and reading only summary sheets shall not be sufficient to answer all the questions.

1 Introduction to the Time Value of Money

Time Value of Money (TVM) is a crucial financial concept that explores the idea that the value of money today is not the same. When we say, **Money has time value,** it simply means that the amount of money you have today is worth more than the same amount of money in the future.

Imagine you have Rs 10,000 with you today. You have two options:

Option 1: You can spend Rs 10,000 right away on something you want.

Option 2: You can choose to invest Rs 10,000 in a bank account that offers an annual interest rate of 5%.

Now, let's see how the time value of money comes into play:

Option 1: If you spend Rs 10,000 today, you'll get immediate satisfaction or use from it, **but your money won't grow.**

Option 2: If you decide to invest the Rs 10,000 in the bank account at a 5% annual simple interest rate, here's what happens:

After one year, your money will grow by 5% (Rs 500), and you'll have Rs 10,500 **Then After two years,** your initial Rs 10,000 will continue to earn the same interest of Rs 500 per year. **So, after two years, you'll have Rs 11,000.**

This is what the basic concept of the Time Value of Money signifies



In finance, it is often said that "money which you hold today is more valuable than the money you hold in future"

This statement essentially means that a **sum of money today is worth more than the same sum of money at a future date**. It highlights the idea that having money now gives you the ability to use it immediately, invest it, or take advantage of opportunities. In contrast, receiving the same amount of money in the future may involve waiting and potentially missing out on these immediate benefits.

Following are the reasons behind the core concept of time value of money.

1.1 Reasons for Time Value of Money

1. **Investment Opportunities** - "Investment Opportunities" means that when you have money today, you can use it to explore various ways to make it grow, **such as investing in stocks, bonds, or even depositing it in a bank account.**

2. **Risk and Uncertainties** - "Risk and Uncertainties" refers to the idea that money available today is generally considered less risky than the same amount of money expected in the future.

3. **Inflation** - Over time, money loses its buying power due to rising prices. Money today can buy more than the same amount in the future.

4. **Consumption:** Individuals generally prefer current consumption to future consumption

Now, we will be covering the techniques of time value of money.

2 Techniques of Time Value of Money

First, let's decode the meaning of Present Value and Future Value

2.1 Present Value and Future Value

1. **Present Value (PV)** measures the current value of an amount of money. Think of it as the money you hold today. PV helps you understand the current worth of a sum of money. **For example**, if you have Rs. 10,000 with you currently and you want to you invest the same in the bank accounts, then Rs. 10,000 is your PV.

2. **Future Value (FV)** measures the Future value of an amount of money. It's like a treasure chest that shows you how much your money can grow over time. Imagine you have Rs. 10,000, and you decide to put it in a special savings bank account. If the account gives you 5% extra each year, FV will tell you that in one year, your Rs. 10,000 will become Rs. 10,500. It's like watching your money grow into a bigger treasure with the help of interest.

Future Value = Present Value + Interest

2.2 Simple and Compound Interest

Interest can be earned in two ways they are simple interest and compound interest method – A- Simple interest, B- Compound Interest

1. **Simple Interest Method**- Simple Interest is a straightforward way of earning or paying interest on money. It's calculated only on the initial sum, known as the principal. **You earn or pay a fixed percentage of the principal**, regardless of how much you've earned or paid in the past.

The formula to calculate Simple Interest (SI) is: **SI = (P * R * T) / 100**

Where:

- **SI stands for Simple Interest.**
- **P is the Principal Amount (the initial sum of money).**
- **R is the Rate of Interest per year (as a percentage)**
- **T is the Time period in years for which interest is calculated.**

2. Compounding Interest Method: Compound Interest is a method of earning or calculating interest on money that involves not only the initial sum (the principal) but also the interest that accumulates over time. It's like your money earns interest, and then that interest also earns more interest.

Imagine you have Rs. 10,000, and you decide to invest it in a savings account with a 10% annual interest rate, compounded annually. The following modus operandi will follow -

- **Year 1**: At the end of the first year, your initial principal of Rs. 10,000 earns 10% interest, which is Rs. 1,000. So, you now have Rs. 11,000.
- **Year 2**: In the second year, you don't just earn interest on the initial Rs. 10,000; you earn it on the new total, **which is Rs. 11,000**. At a 10% rate, you earn Rs. 1,100 in interest. So, you now have Rs. 12,100.
- **Year 3**: In the third year, you continue to earn interest on Rs. 12,100, not just the original Rs. 10,000. At 10%, you earn Rs. 1,210. **So, your total amount to be received becomes Rs. 13,310.**

Here, too we have to remember one basic formula which is **A = P * (1 + R)^(t)**
In this simplified formula:

- A is the future amount, including both principal and interest.
- P is the principal amount (initial sum).
- R is the annual interest rate (in decimal form).
- T is the time in years (here T can also be written as n, both the things mean the same)

Key Observation – Interest earned via compound interest method is bigger as compared to the simple interest method, given the same principal amount, interest rate, and time period.

2.2.1 Compounding Frequency.

1. **Annual Compounding**: If interest is compounded annually, you earn or pay interest on your principal once a year.
2. **Frequent Compounding**: When interest is compounded more frequently, you earn or pay interest multiple times within the year. You can get multiple cases over here, which are discussed as follows
    - **Semi-Annual Compounding** - In semi-annual compounding, interest is added or calculated twice a year, typically every six months. Therefore, the compounding frequency over here becomes 2, which means you will get the interest 2 times in a year.
    - **Quarterly Compounding** - Quarterly compounding involves adding or calculating interest every three months (four times a year). Therefore, the compounding frequency over here becomes 4, which means you will get the interest 4 times in a year.

- **Monthly Compounding**: Monthly compounding means that interest is added or calculated every month (twelve times a year). Therefore, the compounding frequency over here becomes 12, which means you will get the interest 12 times in a year.
- **Continuous Compounding**: Continuous compounding represents the highest frequency, where interest is calculated and added continuously.

So, your new rate of interest will be **R divided by the compounding frequency and T will be multiplied by the compounding frequency.**

Or, alternatively, A = P * (1 + (r/y))^(y*t)

Where, y is the number of times that interest is compounded per year.

Question – Calculate the amount to be received after three years from the following information, assume principal being Rs. 10,000 and rate of interest, being 10% is compounded semiannually, quarterly.

Case 1 - Semi-Annual Compounding - Interest is compounded twice a year, means you will get interest two times in a year, therefore compounding frequency over here is 2. Therefore the rate of interest (R) and your time period (t) values will change. New Rate of Interest will be = Rate of interest/compounding Frequency,

- Wherein existing Rate of Interest is 10%
- Compounding Frequency is 2
- Therefore, new rate of interest will be 10/2 = 5%

New Time Period will be = Time Period * Compounding Frequency

- Wherein existing Time period is 3
- Compounding Frequency is 2
- Therefore, new time period will be 3*2 = 6

Now, you can use the basic formula, A = P * (1 + R)^(T)

- A = 10,000 * (1 + 0.05)^6
- A = 10,000 * (1.05)^6
- A ≈ 13,400

With semi-annual compounding, your Rs. 10,000 investments would grow to approximately Rs. 13,400 after 3 years.

Case 2 - Quarterly Compounding – here Interest is compounded four times a year, so compounding frequency will change to 4. New Rate of Interest will be = Rate of interest/compounding Frequency,

- Wherein Rate of Interest is 10%
- Compounding Frequency is 4
- Therefore, new rate of interest will be 10/4 = 2.5%

New Time Period will be = Time Period * Compounding Frequency

- Wherein Time period is 3
- Compounding Frequency is 4
- Therefore, new time period will be 3*4 = 12

Now, you can use the basic formula, A = P * (1 + R)^(T)

- A = 10,000 * (1 + 0.0.25)^12
- A = 10,000 * (1.025)^12
- A ≈ 13,448.8

Conclusion - More frequent compounding leads to slightly higher amount due to interest being added more often. Students, please see that the same initial **amount can grow to different final amounts depending on how frequently interest is compounded.**

2.2.1.1 Continuous compounding

Continuous compounding is a theoretical concept used in mathematics and finance to model scenarios where interest is calculated and applied continuously. It is a special case of compounding where interest is calculated and added to the principal an infinite number of times

As its, a special case, the formula to calculate amount under **Continuous compounding gets changed, which is A = P * e^(rt)**

Where:

- A is the Future Value (the total amount including both principal and interest).
- P is the Principal Amount (the initial sum of money).
- e is the mathematical constant approximately equal to 2.71 (You have to assume this)
- r is the Annual Interest Rate (in decimal form).
- t is the time the money is invested or borrowed for, in years.

Now, let's move to the concept of compounding and Discounting.

2.3 Compounding and Discounting

Assume that you are investing an amount of Rs. 10,000 in a bank at a 10% annual Compound interest rate for 5 years. Now, Calculate the amount which you will receive after 5 years.

- Principal Amount (P): Rs. 10,000
- Rate of Interest (R): 10% (Compound interest)
- Time (T): 5 years

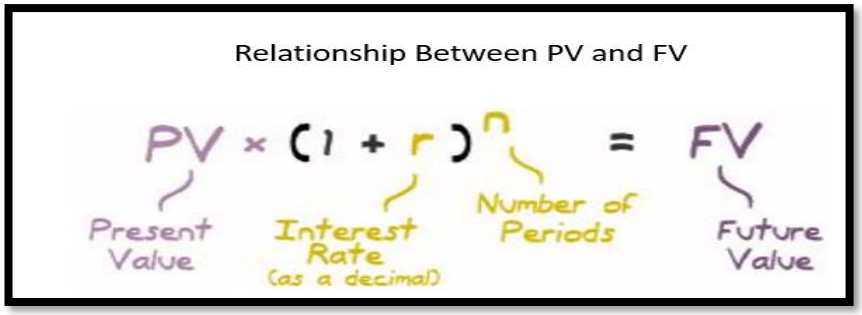
Using the compound Interest formula: **A = P * (1 + R)^T**
Substitute the values:

- A = 10,000 * (1 + 0.10)^5
- A = 10,000 * (1.10)^5
- **Amount ≈ Rs. 16,105.10**

This means, **after 5 years, you will receive, Rs. 16,105 (Approx)** and it will include principal as Rs. 10,000 and another 6,105 will constitute the interest. PV measures the current value of an amount of money, whereas FV measures the Future value of an amount of money

Present Value will be 10,000 and Future Value will be 16,105. Therefore, here we can draw out one big relation between the Present Value and the Future value. **The relation is expressed as follows - FV = PV * (1 + r)^n,** Where:

- **FV stands for Future Value**, which is the value of money at a future point in time.
- **PV represents Present Value**, which is the value of money at the current time or today.
- **r is the annual interest rate** (expressed as a decimal).
- **n is the time period in years**.



NOTE - This formula is exactly the same as we have discussed in the case of compound interest the only difference over here is, that you are denoting Principal as Present Value and Amount as Future Value.

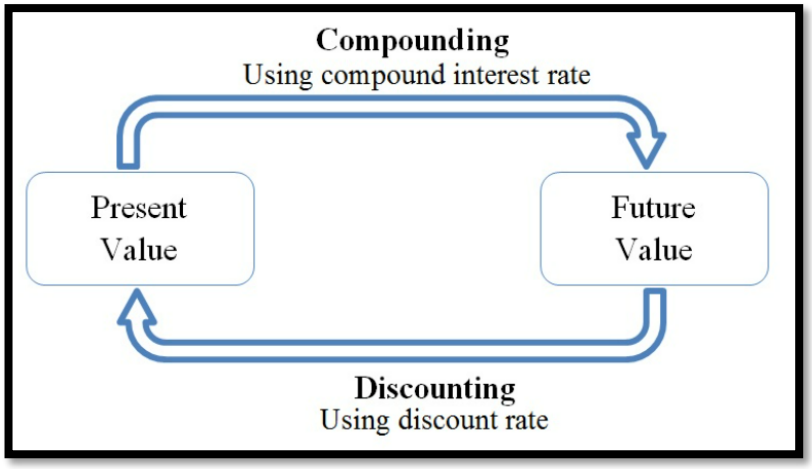
Here's how compounding unfolds:

1. **Starting Point (PV):** You begin with a certain amount of money today. This is your PV, the foundation of your financial journey.
2. **Investment:** You decide to put that money to work. You invest you money and therefore you will start earning interest on it.
3. **Arrival at Future Value (FV):** Eventually, after a certain period of time, you reach your destination, which is the Future Value (FV). This is the total amount you end up with, which includes both your initial principal and all the interest accumulated over time.

Discounting is like the reverse journey of compounding in the world of finance. While compounding takes your money from Present Value (PV) to Future Value (FV), **discounting does the opposite; it takes your money from Future Value (FV) to Present Value (PV)**

Here's how discounting works:

1. **Future Value (FV):** You have a specific amount of money that you expect to receive or spend at some point in the future. This is your FV.
2. **Time Factor:** Money in the future is not as valuable as money today because you could be using it or earning interest on it. So, you "discount" it to find out what it's worth today. We will discuss more about it later on.
3. **Calculating PV:** Discounting helps you find out how much money you need to have today to end up with the FV you expect in the future.



Note – Discount rate in Time Value of Money represents the rate at which future cash flows are adjusted to their present value.

2.3.1 Practical Application of Compounding and Discounting

Question - If you deposit Rs. 25,000 in a bank that offers a 8% annual interest rate on a four-year time deposit, how much would the deposit grow at the end of four years?

To calculate the future value (or the amount), we shall be using our basic formula only, which FV = PV * (1 + r)^t . Now, let's calculate it:

- **FV = 25,000 * (1 + 0.08)^4**
- **FV ≈ 25,000 * 1.36049, FV ≈ Rs. 34,012.25**

So, if you deposit Rs. 25,000 in a bank with an 8% annual interest rate on a four-year time deposit, your deposit will grow to approximately Rs. 34,012.25 at the end of four years. The above example denotes exactly what compounding is all about, you were given the present value, which

was Rs. 25,000 and then you were asked to calculate the Future Value, and I hope you remember, the journey of calculating the future value is **called COMPOUNDING.**

Question on discounting-

Question - You are expecting to receive Rs. 1,000 one year from today. If you want to determine the present value of that amount today, assuming a discount rate of 8%, what is the present value?

The formula remains the same, which is FV = PV * (1 + r)^t **.** Now, let's calculate it:

- PV = 1,000 / (1 + 0.08)
- PV = 1,000 / 1.08
- PV ≈ Rs. 925.93

This is what discounting is all about, in discounting you move backward, that is you start with future value and then arrive at the present value.

Now, lets move to discussion of annuities.

3 Introduction to Annuities

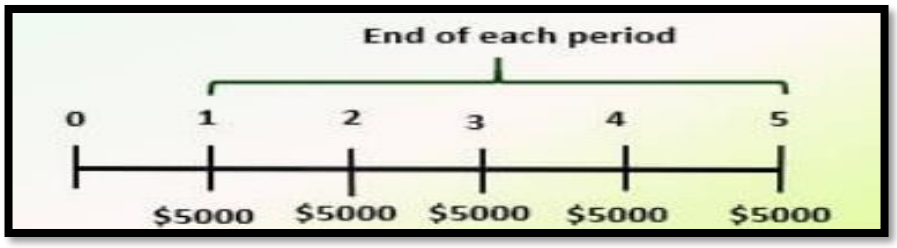
An annuity is like a **series of regular payments or receipt**s that happen at a fixed time interval. Think of it as money coming in or going out regularly, like your monthly rent or your salary.

Example - let's say you win a lottery, and they promise to pay you Rs. 1,000 every month for the next five years. That's an annuity. You'll receive a fixed amount of money regularly for 5 years.

Following are different types of annuities.

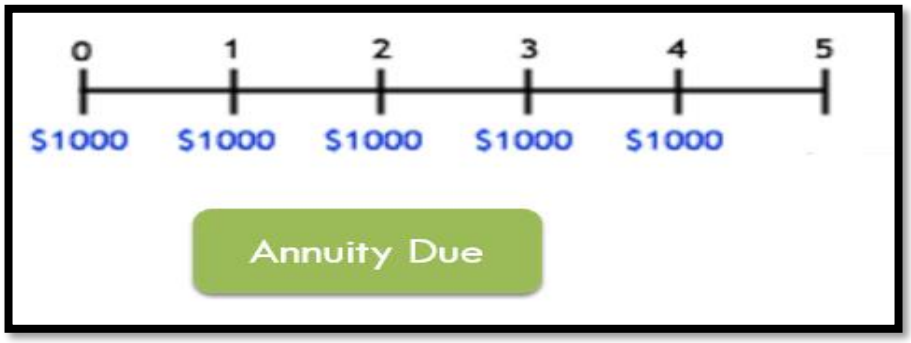
3.1 Types of Annuities

1. **Ordinary Annuity**: An ordinary annuity is a type of annuity where the payments or receipts occur at the end of each period. For example, if you receive your monthly rent of $5000 at the end of each month, it's considered an ordinary annuity. You receive the payment after the time period has elapsed.



2. **Annuity Due** - An annuity due, on the other hand, is a type of annuity where the payments or receipts happen at the beginning of each period. **For instance**, if you receive your monthly

rent of $1000 at the beginning of each month, it's an annuity due because you will receive it at the start of the period.



So, the primary difference between annuity due and ordinary annuity lies in the timing of the amount. In an ordinary annuity, payments/Income occur at the end of each **period, while in an annuity due, payments/income take place at the beginning of each period.**

Now let's proceed ahead to Calculating the Present Values and Future Values of annuities

3.2 Calculating the Present Values and Future Values of annuities

Now, if you remember, we have already learned so as to how to calculate present values and the future values,

Question - You are depositing Rs. 25,000 **"every year"** in a bank that offers a 8% annual interest rate on a four-year time deposit,

Now, if I ask you, is this a case of annuity?

Yes, certainly it is!

Because now you are depositing Rs. 25000 "every year", and remember annuity is just **a series of payments** made at **"equal intervals".**

First, we will learn that, how we can calculate the future values of ordinary annuity and Annuity due.

3.2.1 Future Value of Ordinary Annuity

Question - Suppose you invest **1000 at the end of each year** for next **5** years with interest rate to be **5**%. What would be total value of money at the end of 5 years?

Before, we move to solution, first answer the following questions

Is this an annuity?	Yes, it is, simply because the question says, "at the end of each year", which means there

	is series of amount invested regularly for 5 years
Is this an ordinary annuity?	Yes, it is, the amount is being invested at the end of the year.

We can use the following formula -

$$FV_{Ordinary\ Annuity} = C * \left[\frac{(1+i)^n - 1}{i}\right]$$

C = Cash flow per period
i = interest rate
n = number of payments

As per the question, Cash flow per period = Rs. 5000, Interest Rate = 5% and Number of Payment or Time period = 5.

Just plug in the values now, you will get

- **FV = 1000 [(1+.05)^5–**1] / .05
- FV = 1000 [1.27628 - 1] / .05
- FV = 1000 [.27628] / .05
- FV = 276.28/.05
- FV = 5525.6

Now, let's move to the calculation of the future value of annuity due

3.2.2 Future Value of Annuity Due

Here, we can directly use the short cut method, which is discussed as below –

$$FV\ of\ Annuity\ Due = (1+r) \times P\left[\frac{(1+r)^n - 1}{r}\right]$$

$P = Periodic\ Payment$
$r = rate\ per\ period$
$n = number\ of\ periods$

Now we shall be understanding, so as to how to **calculate the present value of Ordinary Annuity and Annuity Due, here we will be using the discounting techniques.**

3.2.3 Present Value of Ordinary Annuity

Question - What is the present value of the project which pays cash flows of 1000 **at the end of each year** for next **5** years assuming interest rate to be **5**%?

Before, we move to solution, first answer the following questions

Is this an annuity?	Yes, it is, simply because the question says, "at the end of each year"
Is this an ordinary annuity?	Yes, it is, the amount is being invested at the end of the year.
Rs. 1000 is Present Value or Future Value	Obviously, it is the future value given to us.

You can use the following formula directly -

$$PV_{Ordinary\ Annuity} = C * \left[\frac{1-(1+i)^{-n}}{i}\right]$$

- C = Cash flow per period, in the question it is **1000**
- I = interest rate, in the question it is 0.05 or 5%
- n = number of payments, in the question it is 5

Apply the values in the formula, you will get

- PV = 1000 * [(1 - (1+.05) $^{-5}$] / .05
- PV = 4329.47

3.2.4 Present Value Annuity due

For ordinary annuity payments are done at the end of each period. However, for the Annuity Due – Payments are made at the beginning of each period

Here, we can directly use the short cut method, which is discussed as below –

$$PV_{Annuity\ Due} = C * \left[\frac{1-(1+i)^{-n}}{i}\right] * (1+i)$$

But a minute, we still have some small discussions to do. Moving forward now, Let's discuss regarding one interesting topic, which is annuity factors.

4 Present Value and Future Value Factors

To define, Present Value and Future Value Factors, are often referred to **tables which are precomputed values** used in finance to simplify the calculation.

For example, look at the following table

Future Value of 1 Factors (FV of 1 factors)

n	1%	2%	4%	8%	12%
2	1.020	1.040	1.082	1.166	1.254
5	1.051	1.104	1.217	1.469	1.762
10	1.105	1.219	1.480	2.159	3.106
12	1.127	1.268	1.601	2.518	3.896
15	1.161	1.346	1.801	3.172	5.474
16	1.173	1.373	1.873	3.426	6.130
24	1.270	1.608	2.563	6.341	15.179

The above table gives you, which represents the present value of 1 Unit for a combination of years (n) and Interest rates (r). These are also called present Value Interest Factors

Now, you might be thinking that how this table really helps in calculation?

Wait, let solve some to get clarity

Question - If you deposit Rs. 25,000 in a bank that offers 8% annual interest rate, how much would the deposit grow at the end of two years?

Solve this question by yourself first.

Explanation –

If we solve this question, via using the relationship between FV and PV, we will get,

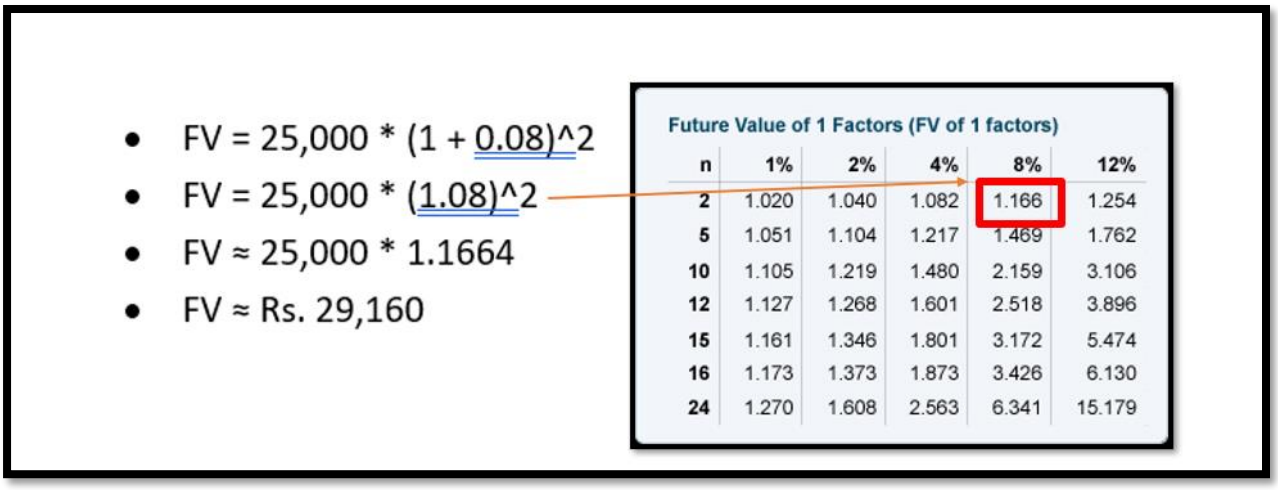
FV = PV * (1 + r)^t

On further solving, Now, let's calculate:

- FV = 25,000 * (1 + 0.08)^2
- FV = 25,000 * (1.08)^2
- FV ≈ 25,000 * 1.1664
- FV ≈ Rs. 29,160

So, if you deposit Rs. 25,000 in a bank with an 8% annual interest rate on a four-year time deposit, the deposit would grow to approximately Rs. **29,160 at the end of two years**.

Now, if you notice above, we have to solve the value of (1.08)^2, the above given factor table, will help you to directly get the value, look at the following image.



All you need to do is, just look **for the interest rate in the columns of the table and look for number of years in the rows of the table**

Now, just some minor discussion is left regarding two interesting topics.

5 Other related concepts

In this section, we will understand two basic concepts -

5.1 Rule of 72, 114 and 144

Rule of 72

By dividing 72 by the annual rate of return, investors can get a rough estimate of how many years

Rule of 114:

To estimate how long it takes to triple your money, divide 114 by your expected interest rate (or rate of return)

Rule of 144: To estimate how long it takes to Quadruple your money, divide 144 by your expected interest rate (or rate of return)

5.2 Effective Interest Rate (EIR)

The Effective Interest Rate (EIR), also known as the Annual Equivalent Rate (AER) or Annual Percentage Yield (APY), is a measure that reflects the true or effective annual interest rate earned or paid on an investment, loan, or financial product, taking into account the effects of compounding.

In simpler terms, the effective interest rate represents the total interest or return you'll receive or pay over the course of a year, factoring in how often interest is added to the principal amount. It provides a more accurate picture of the actual financial impact of a transaction because it considers the frequency of compounding.

Formula to calculate EIR is given as below

$$EIR = (1+r/m)^{m} - 1$$

where EIR = Effective Rate of Interest
r = Nominal Rate of Interest (Yearly Interest Rate)
m = Frequency of compounding per year